Name______________________________ Period __________ Date _____________

GRADE 6
RESOURCE GUIDE
PART 2

MATHLINKS: GRADE 6
RESOURCE GUIDE: PART 2

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STANDARDS FOR MATHEMATICAL PRACTICE

In addition to the mathematical topics you will learn about in this course, your teacher will help you become better at what are called the Mathematical Practices. The Standards for Mathematical Practice describe a variety of processes and strategies to help you to be more mathematically proficient and fluent students.

One way to think about the practices is in groupings.

HABITS OF MIND

MP1 Make sense of problems and persevere in solving them
MP6 Attend to precision

REASONING AND EXPLAINING

MP2 Reason abstractly and quantitatively
MP3 Construct viable arguments and critique the reasoning of others

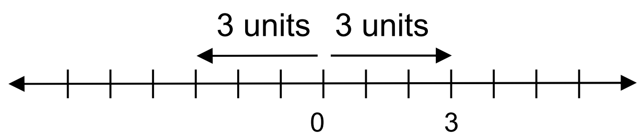
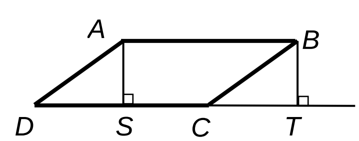
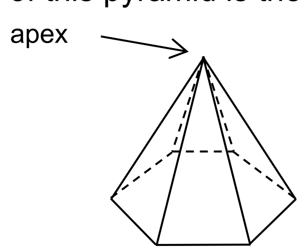
MODELING AND USING TOOLS

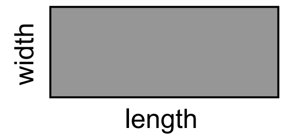
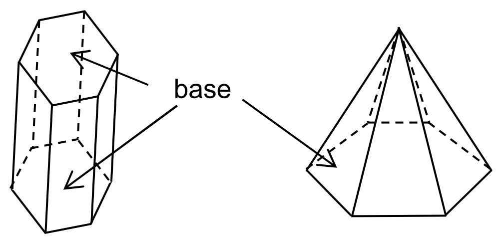
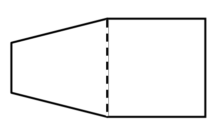
MP4 Model with mathematics
MP5 Use appropriate tools strategically

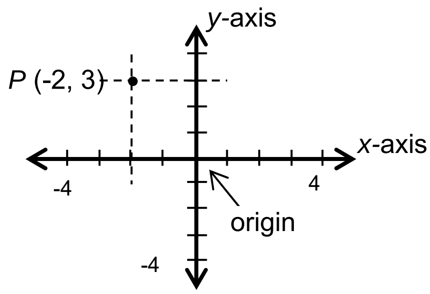
SEEING STRUCTURE AND GENERALIZING

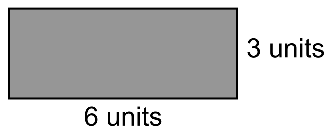
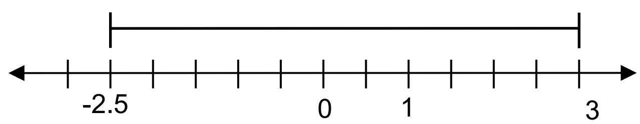
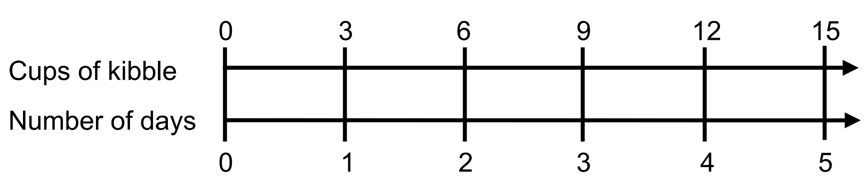
MP7 Look for and make use of structure
MP8 Look for and make use of repeated reasoning

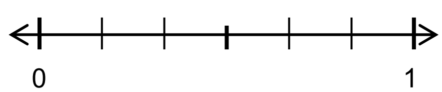
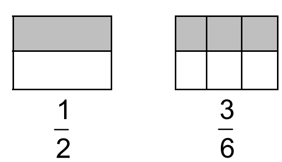
WORD BANK

Word or Phrase	Definition
absolute value	The <u>absolute value</u> $\|x\|$ of a number x is the distance from x to 0 on the number line. Example: $\|3\| = 3$ and $\|-3\| = 3$, because both 3 and -3 are 3 units from 0 on the number line. 
addition property of equality	The <u>addition property of equality</u> states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal. Example: If $4 + 2 = 5 + 1$ and $10 = 2 \bullet 5$ then $4 + 2 + 10 = 5 + 1 + 2 \bullet 5$ check $16 = 16$
altitude of a parallelogram	An <u>altitude of a parallelogram</u> on a given base is a line segment perpendicular to the base and connecting a point on the base (extended if necessary) to a point on the opposite side. The <u>height</u> of the parallelogram is the length of the altitude. Example: In $\square ABCD$, both $\overline{AS}$ and $\overline{BT}$ are altitudes with respect to the base $\overline{CD}$. 
apex	The <u>apex</u> of a solid figure is the highest point of the figure. Example: The apex of this pyramid is the point at the very top. 

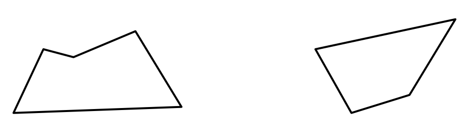
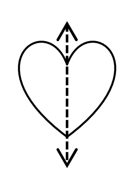
area	The <u>area</u> of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The <u>area of a rectangle</u> is the product of its length and its width.  Area = (length) $\times$ (width) Example: If a rectangle has a length of 12 inches and a width of 5 inches, its area is (5)(12) = 60 square inches.
base	The <u>base</u> of a figure is a predesignated side or face of the figure. The base is usually regarded as the "bottom" of the figure, on which it is standing. The "top" of a figure is sometimes also referred to as a base if it is congruent and parallel to the "bottom." 
coefficient	A <u>coefficient</u> is a number or constant factor in a term of an algebraic expression. Example: In the expression $3x + 5$, 3 is the coefficient of the linear term $3x$, and 5 is the constant coefficient.
composite figure	<u>Composite figure</u> refers to a figure made up of two or more simpler figures. Example: The diagram below shows a composite figure that can be divided into a square and a trapezoid. 
constant term	A <u>constant term</u> in an algebraic expression is a term that has a fixed numerical value. Example: In the expression $5 + 12x^2 - 7$, the terms 5 and -7 are constant terms. If this expression is rewritten as $12x^2 - 2$, the term -2 is the constant term of the new expression.

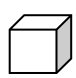
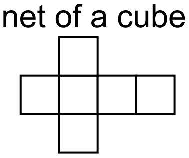
conversion rate	A conversion rate gives the number of units of one measure equal to one unit of another. Example: 60 minutes per 1 hour, or $\frac{60\text{ min}}{1\text{ hr}}$, or 60 $\frac{\text{min}}{\text{hr}}$.
coordinate plane	A (Cartesian) coordinate plane is a plane with two perpendicular number lines (coordinate axes) meeting at a point (the origin). Each point *P* of the coordinate plane corresponds to an ordered pair (*a*, *b*) of numbers, called the coordinates of *P*. The point *P* may be denoted *P*(*a*, *b*). Example: The coordinate axes are often referred to as the *x*-axis and the *y*-axis respectively. The *x*-coordinate -2 of *P* is the number where the line through *P* parallel to the *y*-axis meets the *x*-axis, and the *y*-coordinate 3 of *P* is the number where the line through *P* parallel to the *x*-axis meets the *y*-axis. Points on the *x*-axis have coordinates (-2, 0), and points on the y-axis have coordinates (0,3). The origin has coordinates (0, 0). 
cross-multiplication property	The cross-multiplication property states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. Example: If $\frac{2}{3} = \frac{8}{12}$, then $3 \cdot 8 = 2 \cdot 12$.
customary units	United States customary units are a system of units of measurement that includes ounces, pounds, and tons to measure weight; inches, feet, yards, and miles to measure length; pints, quarts, and gallons to measure capacity; and degrees Fahrenheit to measure temperature. See metric units.
decimal	A decimal is an expression of the form $n.abc\ldots$, where n is a whole number written in standard form, and a, b, c, … are digits. Each decimal represents a unique nonnegative real number and is referred to as a decimal expansion of the number. Example: The decimal expansion of $\frac{4}{3}$ is 1.333333… . Example: The decimal expansion of $\frac{5}{4}$ is 1.25 = 1.2500000… . Example: The decimal expansion of π is non-repeating and non-terminating, but begins with 3.14159… .

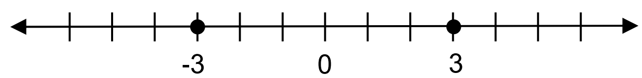
denominator	The denominator of the fraction $\frac{a}{b}$ is b. Example: The denominator of $\frac{3}{7}$ is 7.
dependent variable	A dependent variable is a variable whose value is determined by the values of the independent variables. See independent variable.
dimensions of a rectangle	The dimensions of a rectangle are its length and width. Example: A rectangle of dimensions 6 units by 3 units: 
distance	The distance between two points on a number line is the absolute value of their difference. Example: The distance between 3 and -2.5 is $\lvert 3 - (-2.5) \rvert = \lvert 3 + 2.5 \rvert = 5.5$ or $\lvert -2.5 - 3 \rvert = \lvert -5.5 \rvert = 5.5$. 
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. Example: $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$
double number line diagram	A double number line diagram is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units. Example: The proportional relationship “Bowser eats 3 cups of kibble every 1 day” can be represented in the following double number line diagram. 

equation	An equation is a mathematical statement that asserts the equality of two expressions. When the expressions involve variables, a solution to the equation consists of values for the variables which, when substituted, make the equation true. Example: $5 + 6 = 14 - 3$ is an equation that involves only numbers. Example: $10 + x = 18$ is an equation that involves numbers and a variable. The value for x must be 8 to make this equation true.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. Example: The algebraic expressions $3(x + 4)$ and $3x + 12$ are equivalent. For any value of the variable x, the expressions represent the same number. Example: The numerical expressions $3 + 2$ and $9 - 4$ are equivalent, since both are equal to 5.
equivalent fractions	The fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if they represent the same point on the number line. This occurs if the results of the division problems $a \div b$ and $c \div d$ are equal. Example: Since $\frac{1}{2} = 1 \div 2 = 0.5$ and $\frac{3}{6} = 3 \div 6 = 0.5$, the fractions $\frac{1}{2}$ and $\frac{3}{6}$ are equivalent.  
equivalent ratios	Two ratios are equivalent if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number. Example: 5 : 3 and 20 : 12 are equivalent ratios because both numbers in the ratio 5 : 3 are multiplied by 4 to get to the ratio 20 : 3.

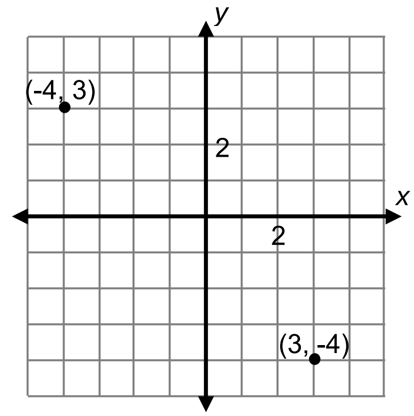
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. Example: To evaluate the expression $3 + 4(5)$, we calculate $3 + 4(5) = 3 + 20 = 23$. Example: To evaluate the expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
exponential notation	The exponential notation b^n (read as " b to the power n ") is used to express n factors of b. The number b is the base, and the number n is the exponent. Example: $2^3 = 2 \cdot 2 \cdot 2 = 8$ The base is 2 and the exponent is 3. Example: $3^2 \cdot 5^3 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = 1{,}125$ The bases are 3 and 5, and the exponents are 2 and 3.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Example: Some mathematical expressions are $7x$, $a + b$, $4v – w$, and 19.
factor of a number	A factor of a number is a divisor of the number. See product. Example: The factors of 12 are 1, 2, 3, 4, 6, and 12.
horizontal	Horizontal refers to being in the same direction as the horizon. The horizontal direction is perpendicular to the force of gravity. Example: On a sheet of lined paper, typically horizontal is the direction that runs left to right. $\longleftrightarrow$
independent variable	An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables. Example: For the function $y = 3x^2 + 1$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.

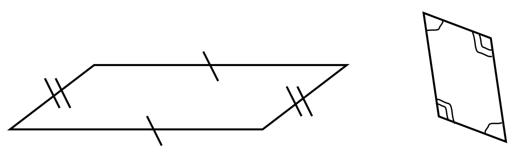
inequality	An inequality is a mathematical statement that asserts the relative size or order of two objects. When the expressions involve variables, a solution to the inequality consists of values for the variables which, when substituted, make the inequality true. Example: $5 > 3$ is an inequality. Example: $x + 3 > 4$ is an inequality. Its solution (which is also an inequality) is $x > 1$.
input-output rule	An input-output rule (or explicit rule) is a rule that establishes explicitly an output value for each given input value. Example: The equation $y = x + 3$ corresponds to the explicit rule "output = input + 3." If the input (x) is 17, then the output (y) is $17 + 3 = 20$.
integers	The integers are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … .
inverse operation	The inverse operation to a mathematical operation reverses the effect of the operation. Example: Addition and subtraction are inverse operations. Example: Multiplication and division are inverse operations.
irregular polygon	An irregular polygon is a polygon that is not regular. See regular polygon. 
like terms	Terms of a mathematical expression that have the same variable part are referred to as like terms. See term. Example: In the mathematical expression $2x + 6 + 3x + 5$, the terms $2x$ and $3x$ are like terms, and the terms 6 and 5 are like terms.
line of symmetry	A line of symmetry is a line that divides a figure into two parts that are reflections of each other across the line. Example: The heart-shape figure to the right has a vertical line of symmetry. 

metric units	Metric units are a system of units of measurement that includes grams and kilograms to measure weight; millimeters, centimeters, meters, and kilometers to measure length; milliliters and liters to measure capacity; and degrees Celsius to measure temperature. See customary units.
mixed number	A mixed number is an expression of the form $n\frac{p}{q}$, which is a shorthand for $n + \frac{p}{q}$, where n, p, and q are positive whole numbers and $p < q$. Example: The mixed number $4\frac{1}{4}$ ("four and one fourth") is shorthand for $4 + \frac{1}{4}$. It should not be confused with the product $4 \bullet \frac{1}{4} = 1$.
multiplication property of equality	The multiplication property of equality states that if $a = b$ and $c = d$, then $ac = bd$. In other words, equals multiplied by equals are equal. Example: If $4 + 2 = 5 + 1$ and $10 = 2 \bullet 5$ then $10 \bullet (4 + 2) = 2 \bullet 5 \bullet (5 + 1)$ Check $60 = 60$
net	A net for a three-dimensional figure is a two-dimensional pattern for the figure. When cut from a sheet of paper, a net forms one connected piece which can be folded and the edges joined to form the given figure. Example: cube  net of a cube 

numerical coefficient	The numerical coefficient of a term in an algebraic expression is the numerical factor of the term. See coefficient. Example: The numerical coefficient of $6x^2$ is 6. Example: The numerical coefficient of $-x$ is -1. Example: The numerical coefficient of $(x + 2) \bullet 3$ is 3.
opposite of a number	The opposite of a number n, written $-n$, is its additive inverse. Algebraically, the sum of a number and its opposite is zero. Geometrically, the opposite of a number is the number on the other side of zero at the same distance from zero. Example: The opposite of 3 is -3, because $3 + (-3) = -3 + 3 = 0$. Example: The opposite of -3 is $-(-3) = 3$.  Thus the opposite of a number does not have to be negative.
order of operations	An order of operations is a convention, or set of rules, that specifies in what order to perform the operations in an algebraic expression. The standard order of operations is as follows: 1. Do the operations in grouping symbols first. (Use rules 2-4 inside the grouping symbols.) 2. Calculate all the expressions with exponents. 3. Multiply and divide in order from left to right. 4. Add and subtract in order from left to right. In particular, multiplications and divisions are carried out before additions and subtractions. Example: $\frac{3^2+(6\bullet 2-1)}{5} = \frac{3^2+(12-1)}{5} = \frac{3^2+(11)}{5}$ $= \frac{9+(11)}{5} = \frac{20}{5} = 4$

<table>
<tr><td>ordered pair</td><td>An ordered pair of numbers is a pair of numbers with a specified order. Ordered pairs are denoted (a, b), (x, y), (s, t), etc.

Example: Ordered pairs of numbers are used to represent points in a coordinate plane. The ordered pair (3, -4) represents the point with x-coordinate 3 and y-coordinate -4. This is different from the ordered pair (-4, 3).


</td></tr>
<tr><td>origin</td><td>The origin of a coordinate plane is the point (0, 0) where the vertical and horizontal coordinate axes intersect. See coordinate plane.</td></tr>
<tr><td>parallelogram</td><td>A parallelogram is a quadrilateral in which opposite sides are parallel. In a parallelogram, opposite sides have equal length and opposite angles have equal measure.

</td></tr>
<tr><td>percent</td><td>A percent is a number expressed in terms of the unit $1\% = \frac{1}{100}$. To convert a positive number to a percent, multiply the number by 100. To convert a percent to a number, divide the percent by 100.

Example: Fifteen percent $= 15\% = \frac{15}{100} = 0.15$

Example: $4 = 4 \times 100\% = 400\%$</td></tr>
<tr><td>percent of a number</td><td>A percent of a number is the product of the percent and the number. It represents the number of parts per 100 parts.

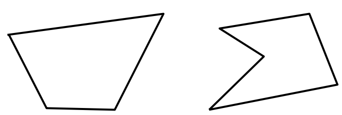
Example: 15% of 300 is $\frac{15}{100} \bullet 300 = 45$.

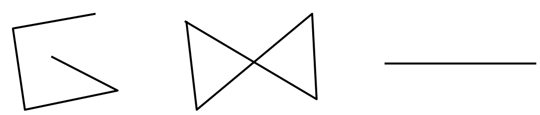
Example: If 45 out of 300 students are boys, then 15 out of every 100 students are boys, and 15% of the students are boys.</td></tr>
</table>

<table>
<tr><td>perimeter</td><td>The perimeter of a plane figure is the length of the boundary of the figure.

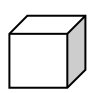
Example: The perimeter of a square is four times its side-length.

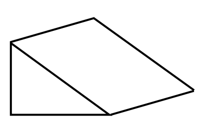
Example: The perimeter of a rectangle is twice the length plus twice the width.</td></tr>
<tr><td>plane</td><td>A plane is a flat two-dimensional surface without holes that extends to infinity in all directions.</td></tr>
<tr><td>polygon</td><td>A polygon is a special kind of figure in a plane made up of a chain of line segments laid end-to-end to enclose a region. Each endpoint of a segment of the polygon meets the endpoint of one other segment, otherwise the segments do not meet each other. The line segments are the sides (or edges) of the polygon, and the endpoints of the line segments are the vertices of the polygon. A polygon divides the plane into two regions, an “inside” and an “outside.” The region inside a polygon may also be referred to as a polygon.

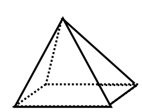

polygons

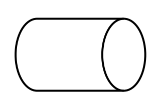

not polygons</td></tr>
<tr><td>polyhedron</td><td>A polyhedron is a closed figure in three-dimensional space consisting of a finite number of polygons that are joined at their edges and that form the boundary of the enclosed solid figure. The polygons are the faces of the polyhedron, the edges of the polygons are the edges of the polyhedron, and the vertices of the polygons are the vertices of the polyhedron. A polyhedron divides space into two regions, an “inside” and an “outside.” The region inside a polyhedron may also be referred to as a polyhedron.

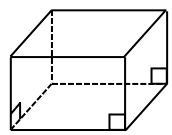
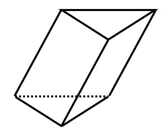
Examples: A cube is a polyhedron. It has 6 faces, 12 edges, and 8 vertices. A cylinder is not a polyhedron.

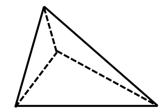
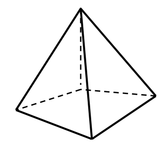
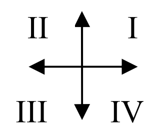

polyhedron (cube)

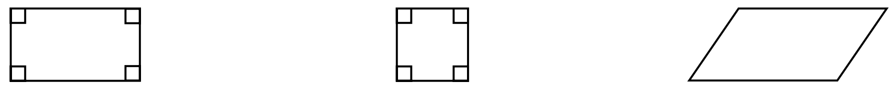
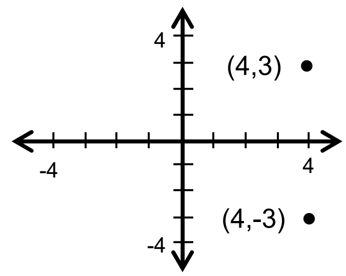
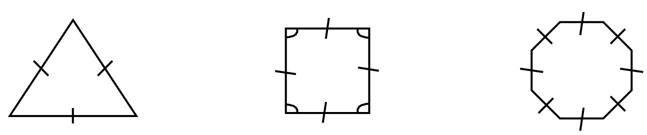
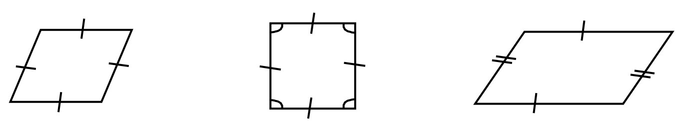
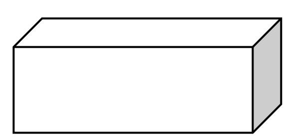

polyhedron (triangular prism)


polyhedron (square pyramid)


not a polyhedron (cylinder)</td></tr>
</table>

prism	A prism is a polyhedron in which two faces (the bases) are congruent parallel polygons, and the other faces (the lateral faces) are parallelograms. If the lateral faces are perpendicular to the bases, the prism is a right prism. Otherwise, the prism is an oblique prism. Example: A right rectangular prism is a right prism whose bases are rectangles and faces are rectangles. Example: An oblique triangular prism is a prism whose bases are triangles and faces are parallelograms.  right rectangular prism  oblique triangular prism
product	A product is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product. Example: The product of 7 and 8 is 56, written $7 \bullet 8 = 56$. The numbers 7 and 8 are both factors of 56.
proportional	Two quantities are proportional if one is a multiple of the other. We say that y is proportional to x if $y = kx$, where k is the constant of proportionality. Example: If Bowser eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.
proportional relationship	Two variables are in a proportional relationship if the values of one are the same constant multiple of the values of the other. The constant is referred to as the constant of proportionality. Example: If Bowser eats 3 cups of kibble each day, then the number of cups of kibble and the number of days are in a proportional relationship.

pyramid	A pyramid is a polyhedron in which one face (the base) is a polygon, and the other faces are triangles with a common vertex (the apex). Each edge of the base is the side of a triangular face with the opposite vertex at the apex. Example: A tetrahedron is a pyramid with a triangular base.  Example: A square pyramid is a pyramid with a square base. The Egyptian pyramids are square pyramids, as they have square bases. 
quadrants	The coordinate axes of a coordinate plane separate the plane into four regions, called quadrants. The quadrants are labeled $\mathrm{I} - \mathrm{IV}$ starting from the upper right region and going counterclockwise.  Example: The point (8, 5) is located in quadrant I, while (-3, -5) is located in quadrant III.
rate	See unit rate.
ratio	A ratio is a pair of nonnegative numbers, not both zero, in a specific order. The ratio of a to b is denoted by $a : b$ (read "*a* to *b*," or "*a* for every *b*"). Example: The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. Use 3 cups of water for every 2 cups of juice.
rational number	A rational number is a number that can be expressed as a quotient of integers. Example: $\frac{3}{5}$ is rational because it is a quotient of integers. Example: $2\frac{1}{3}$ and 0.7 are rational numbers because they **can be** expressed as quotients of integers, $2\frac{1}{3} = \frac{7}{3}$ and $0.7 = \frac{7}{10}$. Example: $\sqrt{2}$ and π are not rational numbers. They **cannot be** expressed as a quotient of integers.

rectangle	A rectangle is a quadrilateral with four right angles. In a rectangle, opposite sides are parallel and have equal length. Example: A square is a rectangle with four congruent sides.  rectangle square not a rectangle
reflection	The reflection of a plane through a line refers to the transformation that takes a point on one side of the line to its mirror image on the other side of the line. Example: When the plane is reflected through the *x*-axis, the point (4,3) is taken to the point (4,-3). 
regular polygon	A regular polygon is a polygon whose sides are all congruent, and whose angles are all congruent. The sides of a regular polygon have the same length, and the angles have the same angle measure. Example: A regular triangle is an equilateral triangle, and a regular quadrilateral is a square. A stop sign has the shape of a regular octagon. 
rhombus	A rhombus is a parallelogram whose sides have equal length.  is a rhombus is a rhombus not a rhombus
right rectangular prism	A right rectangular prism is a six-sided polyhedron in which all the faces are rectangles. The opposite faces of a right rectangular prism are parallel to each other. The distances between pairs of opposite faces are the length, width, and height of the right rectangular prism. Example: A rectangular box is a right rectangular prism. 

<table>
<tr><td>scale factor</td><td>A <u>scale factor</u> is a positive number which multiplies some quantity.

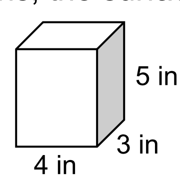
Example: To make a scale drawing of a figure, we multiply all lengths by the same scale factor. If the scale factor is greater than 1, the figure is expanded, and if the scale factor is between 0 and 1, the figure is reduced in size.</td></tr>
<tr><td>solution to an equation</td><td>A <u>solution to an equation</u> involving variables consists of values for the variables which, when substituted, make the equation true.

Example: The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.</td></tr>
<tr><td>solve an equation</td><td>To <u>solve an equation</u> refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called <u>solutions</u> to the equation.

Example: To solve the equation $2x = 6$, one might think “two times what number is equal to 6?” The only value for x that satisfies this condition is 3, which is then the solution.</td></tr>
<tr><td>square number</td><td>A <u>square number</u> is a number that is the product of a positive integer and itself.

Example: 9 is a square number, since $3 \times 3 = 9$.</td></tr>
<tr><td>substitution</td><td><u>Substitution</u> refers to replacing a value or quantity with an equivalent value or quantity.

Example: If $x + y = 10$, and $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.</td></tr>
<tr><td>surface area</td><td>The <u>surface area</u> of a three-dimensional figure is a measure of the size of the surface of the figure, expressed in square units. If the surface of the three-dimensional figure consists of two-dimensional polygons, the surface area is the sum of the areas of the polygons.


Example: A rectangular box has a length of 3 in, a width of 4 in, and a height of 5 in.

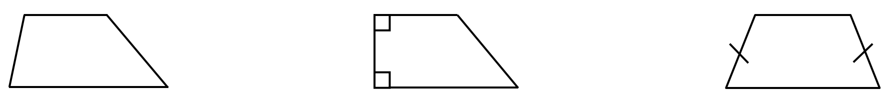
Surface Area $= 2(3 \bullet 4) + 2(3 \bullet 5) + 2(4 \bullet 5)$
$= 94$ square inches</td></tr>
</table>

<table>
<tr><td>tape diagram</td><td>
A tape diagram is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities.

Example: This tape diagram represents a drink mixture with 3 parts grape juice for every 2 parts water.

<table><tr><td>G</td><td>G</td><td>G</td><td>W</td><td>W</td></tr></table>
</td></tr>
<tr><td>term</td><td>
The terms in a mathematical expression involving addition (or subtraction) are the quantities being added (or subtracted). Terms that have the same variable part are referred to as like terms.

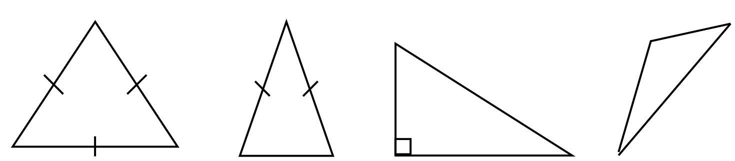
Example: The expression $2x + 6 + 3x + 5$ has four terms: $2x$, 6, $3x$, and 5. The terms $2x$ and $3x$ are like terms, since each is a constant multiple of x. The terms 6 and 5 are like terms, since each is a constant.
</td></tr>
<tr><td>trapezoid</td><td>
A trapezoid is a quadrilateral with at least one pair of parallel sides. A right trapezoid is a trapezoid with two adjacent right angles. An isosceles trapezoid is a trapezoid in which non-parallel sides are congruent.



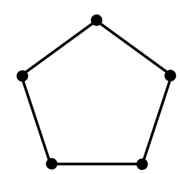
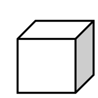
trapezoid right trapezoid isosceles trapezoid
</td></tr>
<tr><td>triangle</td><td>
A triangle is a three-sided polygon. Triangles may be classified by their sides or their angles.

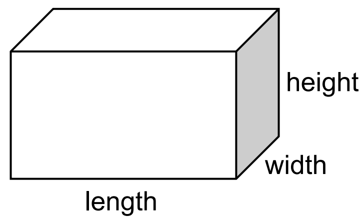
If the three sides of the triangle have the same length, it is an equilateral triangle. If at least two sides have the same length, it is an isosceles triangle. If no two sides have the same length, it is a scalene triangle.

If the three angles of a triangle have the same measure, it is an equiangular triangle. If all angles of the triangle are less than 90°, it is an acute triangle. If one of the angles of the triangle equals 90°, it is a right triangle. If one of the angles of the triangle is greater than 90°, it is an obtuse triangle.



<table>
<tr><td>classified by sides:</td><td>Equilateral</td><td>Isosceles</td><td>Scalene</td><td>scalene</td></tr>
<tr><td>classified by angles:</td><td>equiangular</td><td>acute</td><td>right</td><td>obtuse</td></tr>
</table>
</td></tr>
</table>

unit price	A <u>unit price</u> is a price for one unit of measure. Example: If 4 apples cost \$1.00, then the unit price is \$0.25 for one apple, or 0.25 dollars per apple.
unit rate	The <u>unit rate</u> associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. Example: The ratio of 40 miles each 5 hours has unit rate 8 miles per hour.
variable	A <u>variable</u> is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to inputs and outputs of functions, to quantities that vary in a relationship, or to unknown quantities in equations and inequalities. Example: In the equation $d = r\,t$, the quantities d, r, and t are variables. Example: In the equation $6 = 2x + 8$, the variable x may be referred to as the unknown.
variable expression	See <u>expression</u>.
vertex	A <u>vertex</u> of a polygon or polyhedron is a point where two edges meet. See <u>polygon</u>, <u>polyhedron</u>. Example: A pentagon has five vertices. 
vertical	<u>Vertical</u> refers to being in the same direction as the force of gravity. The vertical direction is perpendicular to the horizontal direction. Example: On a sheet of lined paper, typically the vertical direction runs up and down. 
volume	The <u>volume</u> of a three-dimensional figure is a measure of the size of the figure, expressed in cubic units. The volume of a right rectangular prism is the product of its length, width, and height. The volume of a union of non-overlapping figures is the sum of their volumes. Example: The volume of a cube with side length of 3 units is $3 \cdot 3 \cdot 3 = 27$ cubic units. 

volume of a right rectangular prism	The volume of a right rectangular prism is the product of its length, width, and height. See right rectangular prism.  Volume = (length) • (width) • (height)
whole numbers	The whole numbers are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, … .
x-axis	The *x*-axis is the horizontal number line passing through the origin in a coordinate plane. See coordinate plane.
x-coordinate	See coordinate plane.
y-axis	The *y*-axis is the vertical number line passing through the origin in a coordinate plane. See coordinate plane.
y-coordinate	See coordinate plane.

MATHEMATICAL PROPERTIES

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).

These include:

- Associative property of addition
- Commutative property of addition
- Additive identity property
- Additive inverse property
- Associative property of multiplication
- Commutative property of multiplication
- Multiplicative identity property
- Multiplicative inverse property
- Distributive property relating multiplication and addition

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences). These include:

- Addition property of equality
- Multiplication property of equality

ORDER OF OPERATIONS

Order of Operations

There are many mathematical conventions that enable us to interpret mathematical notation and to communicate efficiently. The agreed-upon rules for interpreting mathematical notation, important for simplifying arithmetic and algebraic expressions, are called the standard order of operations.

Step 1: Do the operations in grouping symbols first (e.g. use rules 2-4 inside parentheses).
Step 2: Calculate all the expressions with exponents.
Step 3: Multiply and divide in order from left to right.
Step 4: Add and subtract in order from left to right.

Example:

$$\frac{3^2 + (6 \bullet 2 - 1)}{5} = \frac{3^2 + (12 - 1)}{5} = \frac{3^2 + (11)}{5} = \frac{9 + (11)}{5} = \frac{20}{5} = 4$$

There are many times when these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for \$1.50 each and 3 bags of peanuts for \$1.25 each. Write an expression for this situation, and simplify the expression to find the total cost.

Expression: $\underbrace{2 \bullet (1.50)}_{3.00} + \underbrace{3 \bullet (1.25)}_{3.75}$

$$3.00 + 3.75 = 6.75$$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

However, if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

$$2\,(1.50) = 3 \quad \rightarrow \quad 3 + 3 = 6 \quad \rightarrow \quad 6\,(1.25) = 7.50$$

Using Order of Operations to Simplify Expressions		
Order of Operations	**Expression**	**Comments**
	$\dfrac{2^3 \div 2\,(5-2)}{4 + 2 \bullet 10}$	
1. Simplify expressions within grouping symbols.	$\dfrac{2^3 \div 2\,(3)}{4 + 2 \bullet 10}$	Parentheses are grouping symbols: Therefore $5 - 2 = 3$. The fraction bar is also a grouping symbol, so the first step here is to simplify the numerator and denominator.
2. Calculate powers.	$\dfrac{8 \div 2\,(3)}{4 + 2 \bullet 10}$	$2^3 = 2 \bullet 2 \bullet 2 = 8$
3. Perform multiplication and division from left to right.	$\dfrac{12}{4 + 20}$	In the numerator: Divide 8 by 2, then multiply by 3. In the denominator: Multiply 2 by 10.
4. Perform addition and subtraction from left to right.	$\dfrac{12}{24} = \dfrac{1}{2}$	Perform the addition: $4 + 20 = 24$. Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed.

EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Variables in Algebra

Loosely speaking, variables are quantities that can vary. Variables are represented by letters or symbols.

Variables have many different uses in mathematics. The use of variables, together with the rules of arithmetic, makes algebra a powerful tool.

Three important ways that variables appear in algebra are the following.

Usage	Examples
Variables can represent an <u>unknown quantity</u> in an equation or inequality. In this case, the equation is valid only for specific value(s) of the variable.	$x + 4 = 9$ $5x = 20$ $1 + 2x < 10$
Variables can represent <u>quantities that vary</u> in a relationship. In this case, there is always more than one variable in the equation.	Formula: $P = 2\ell + 2w$, $A = s^2$ Function (input-output rule): $y = 5x$, $y = x + 3$
Variables can represent <u>quantities in statements that generalize</u> rules of arithmetic, quantities in identities (statements that are always true), or quantities in inequalities that follow from rules of arithmetic. In this case, there may be one or more variables.	Commutative property of addition: $x + y = y + x$ Distributive property: $x(y + z) = xy + xz$ Multiplicative identity: $x(\frac{1}{x}) = 1$ for $x \neq 0$ Identities: $5(x + 2) = 5x + 10$ Inequalities: $x < x + 1$

Writing Expressions

The notation used for algebra is sometimes different from the notation used for arithmetic. For example:

- 54 means the sum of five tens and four ones, that is, $5(10) + 4$.
- $5\frac{1}{2}$ means the sum of five and one-half. that is, $5 + \frac{1}{2}$.
- $5x$ means the product of 5 and x, which can also be written $5(x)$ or $5 \bullet x$. We typically do not write $5 \times x$ because the multiplication symbol '$\times$' is easily confused with the variable x.

Evaluate or Simplify?

We use the word "evaluate" when we want to calculate the value of an expression.

Example: To evaluate $16 - 4(2)$, follow the rules for order of operations and compute: $16 - 4(2) = 16 - 8 = 8$.

To evaluate $6 + 3x$ when $x = 2$, substitute 2 for x and calculate: $6 + 3(2) = 6 + 6 = 12$.

We use the word "simplify" when rewriting a number or an expression in a form more easily readable or understandable.

Example: To simplify $2x + 3 + 5x$, combine like terms: $2x + 3 + 5x = 7x + 3$.

Sometimes it may not be clear what the simplest form of an expression is. For instance, by the distributive property, $4(x + 2) = 4x + 8$. For some applications, $4(x + 2)$ may be considered simpler than $4x + 8$, but for other applications, $4x + 8$ may be considered simpler than $4(x + 2)$.

Equality and Inequality

Here are some symbols for equality and inequality. In some sense, they behave as mathematical "verbs" because they connect two phrases (expressions) to make a sentence.

Symbol	Word phrase	Examples
=	is equal to	$5 + 3 = 2 + 6$
≤	is less than or equal to	$3 - 1 \leq 10 + 5$ $3 - 1 \leq 2 + 0$
≥	is greater than or equal to	$-5 \geq 6 - 14$ $-5 \geq 6 - 11$
<	is less than	$0.33 < \frac{1}{3}$
>	is greater than	$0.55505 > 0.55$
≠	not equal to	$2 + 5 \neq 25$

How to Determine if an Equation or Inequality is True

PIZZA SHOP MENU (The variable represents the cost of an item.)			
Pizza		**Drinks**	
Cheese slice (c)	$1.00	Small drink (s)	$0.95
Pepperoni slice (p)	$1.25	Large drink (L)	$1.60

1. What value makes this equation true? $p + \square = 3c$

Substitute: $1.25 + \square = 3(1.00)$

$1.25 + 1.00 = 3.00$? NO

$1.25 + 1.25 = 3.00$? NO

$1.25 + 0.95 = 3.00$? NO

$1.25 + 1.75 = 3.00$? YES

The equation is true when $\square$ represents the cost of a large drink ($L = 1.75$).

2. Determine whether each inequality is true or false.

Is $c + p > L$?	Is $s + c < p$?	Is $5c > 4p$?
Is $1.00 + 1.25 > 1.75$?	Is $0.95 + 1.0 < 1.25$?	Is $5(1.00) > 4(1.25)$?
Is $2.25 > 1.75$?	Is $1.95 < 1.25$?	Is $5.00 > 5.00$?
true	false	false

Solving Equations Using Mental Math

To solve an equation using mental math, apply your knowledge of arithmetic facts to find value(s) that make the equation true.

Example 1: Solve $3x = 15$

Think. "What number multiplied by 3 is 15?
$3(5) = 15$.
Therefore $x = 5$."

Example 2: Solve $12 = 20 - k$.

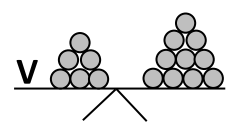
Think. "$12 = 20 - k$ can be thought of as $20 - k = 12$.
20 – "something" equals 12.
$20 - 8 = 12$.
Therefore $k = 8$.
I know this is right (check) because $20 - 8 = 12$."

Solving Equations Using Balance Strategies

An equal sign signifies that two expressions have the same value. A balance scale illustrates this concept.

On the scales below, **V** represents a cup with an unknown number of marbles in it.

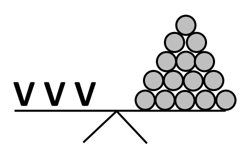
Example 1: Suppose 1 cup and 6 marbles balance 10 marbles.
How many marbles must be in the cup?



Remove 6 marbles from each side of the balance scale.
It will still balance.
1 cup will balance 4 marbles.
Therefore the cup holds 4 marbles.

This balance scale illustrates how to solve the equation $x + 6 = 10$.

Example 2: Suppose 3 cups with the same amount of marbles in each balance 15 marbles. How many marbles must be in each cup?



Divide the marbles into 3 equal parts.
3 cups balance 3 piles of 5 marbles each.
Each cup must balance 5 marbles.

This balance scale illustrates how to solve the equation $3x = 15$.

Solving Equations Using Doing and Undoing

Since addition and subtraction are inverse operations, we can undo addition with subtraction and vice versa.

Since multiplication and division are inverse operations, we can undo multiplication with division and vice versa.

Example 1: Solve: $x - 28 = 50$

Think: "I see that a number minus 28 is 50. I can undo subtraction with addition. Therefore, the unknown number must be 50 + 28 = 78."

Write left to right: x → subtract 28 → 50

Write right to left: 78 ← add 28 ← 50

Check: 78 – 28 = 50

Example 2: Solve: $\frac{x}{8} = 14$

Think: "I see that a number divided by 8 is 14. I can undo division with multiplication. Therefore the unknown number must be 14 times 8."

Write left to right: x → divided by 8 → 14

Write right to left: 112 ← multiplied by 8 ← 14

Check: $\frac{112}{8} = 14$

Similar Phrases with Different Meanings

Sometimes it is useful to "translate" words into symbols. Here are some tips.

Example 1: "Is less than" versus "less than"
"4 is less than 10" translates to the <u>inequality</u> $4 < 10$.
"4 less than 10" translates to the <u>expression</u> $10 - 4$.

Example 2: "Is greater than" verses "greater than"
"7 is greater than 2 + 3" translates to the <u>inequality</u> $7 > 2 + 3$.
"7 greater than 2 + 3" translates to the <u>expression</u> $(2 + 3) + 7$.

One way you may be able to distinguish between these ideas is to think about the parts of speech in English. "Is greater than" behaves like a mathematical verb. "Greater than" is a phrase. In English, we connect phrases with verbs to make sentences. The same is true in mathematics.

RATIOS AND PROPORTIONAL RELATIONSHIPS

Ratios: Language and Notation

The ratio of a to b is denoted by $a : b$ (read "a to b," or "a for every b").

Note that the ratio of a to b is not the same as the ratio of b to a unless $a = b$.

We can identify several ratios for the objects in the picture to the right. ○○○☆☆

- There are 3 circles for every 2 stars.
- The ratio of stars to circles is 2 to 3.
- The ratio of circles to total shapes is 3 : 5
- The ratio of circles to stars is 3 : 2.

If we make 3 copies of the figure above, we get the picture to the right, in which the ratio of circles to stars is 9 : 6. The ratio 9 : 6 is obtained by multiplying each number in the ratio 3 : 2 by 3. In each of the pictures, there are $\frac{3}{2}$ as many circles as there are stars, or if we imagine distributing them, there are $\frac{3}{2}$ = 1.5 circles for every star . We refer to 1.5 as the <u>value of the ratio</u> or the <u>unit rate</u>.

○○○☆☆
○○○☆☆
○○○☆☆

If both numbers in one ratio are multiplied by the same positive number, we arrive at an equivalent ratio. The arrow diagram to the right illustrates that the ratio 3 : 2 is equivalent to the ratio 9 : 6. We will call the number 3 on the sides of the arrow diagram the "multiplier." Equivalent ratios have the same value.

$$\begin{array}{ccc} 3 & : & 2 \\ \times 3 \downarrow & & \downarrow \times 3 \\ 9 & : & 6 \end{array}$$

Tables of Number Pairs

Tables are useful for recording number pairs that have equivalent ratios. In the case of a ratio of three circles for every two stars, there are two ways that number pairs with equivalent ratios might be recorded in a table.

○○○☆☆

Table 1 is aligned horizontally, and the number pairs of circles and stars are placed in columns. The ratios represented by the columm pairs are all equivalent since they all have the same value of 1.5.

Table 1

Circles	3	6	9
Stars	2	4	6

Table 2 is aligned vertically, and the number pairs of circles and stars are placed in rows. The ratios represented by the row pairs are all equivalent, since they all have the same value of 1.5.

Table 2

Circles	**Stars**
3	2
6	4
9	6

Tape Diagrams

A tape diagram is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities. Tape diagrams are typically used when quantities have the same units.

Here are two versions of tape diagrams that show that the ratio of grape juice to water in some mixture is 2 : 4.

Diagram 1

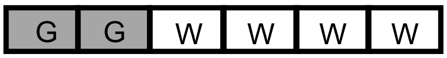
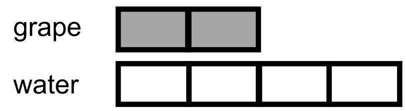
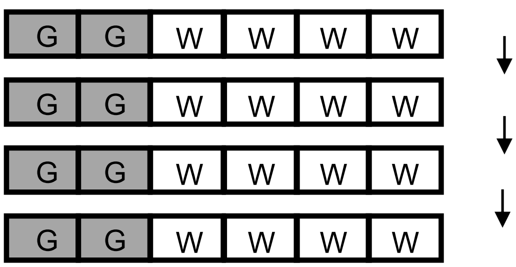


Diagram 2



Suppose we want to know how much grape juice is needed to make a mixture that is 24 gallons. Here are two methods using Diagram 1.

Method 1:



2 gal grape : 4 gal water → 6 gal total (not enough)

4 gal grape : 8 gal water → 12 gal total (not enough)

6 gal grape : 12 gal water → 18 gal total (not enough)

8 gal grape : 16 gal water → 24 gal total (perfect)

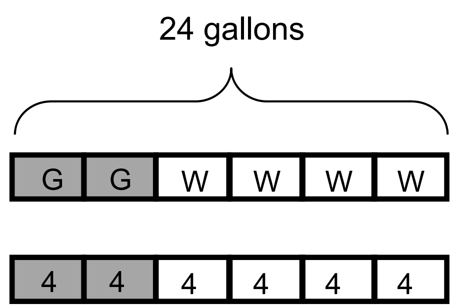
24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents 1 unit (1 gallon of liquid.)

Method 2:

Six rectangles in the tape diagram represent 24 gallons of mixture.

Therefore, one rectangle in the tape diagram represents 4 gallons of grape juice.



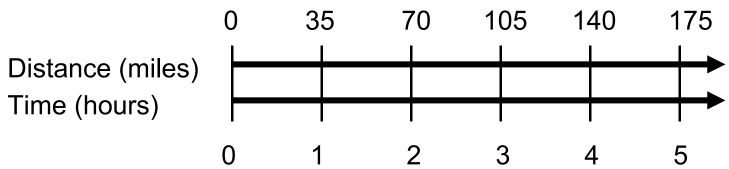
24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents more than 1 unit (4 gallons, in this case). Pieces of tape in the diagrams do not always need to represent 1 unit.

Double Number Line Diagrams

A double number line diagram is a graphical representation of two quantities, in which corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used to compare two quantities that have different units.

The double number line below shows corresponding ratios if a car goes 70 miles every 2 hours.



We can see from the double number line diagram above that at the given rate, the car goes 35 miles in 1 hour, 105 miles in 3 hours, etc. Notice here, the same tick marks on the number line are used to represent different quantities.

Unit Rate and Unit Price

The unit rate associated with a ratio is the value of the ratio, to which we usually attach units for clarity. In other words, the unit rate associated with the ratio $a : b$ is the number $\frac{a}{b}$, to which we may attach units. For this to make sense, we must assume that $b \neq 0$.

Example: A car goes 70 miles for every 2 hours.

- This may be represented by the ratio 70 : 2.
- The number $\frac{70}{2} = \frac{35}{1} = 35$ is the value of the ratio.
- The unit rate is then the value 35, to which we attach the units "miles per hour." Thus the unit rate can be written in either of the forms below.

$$35\ \frac{\text{miles}}{\text{hour}} \qquad \text{or} \qquad 35 \text{ miles per hour}$$

A unit price is the price for one unit.

Example: It costs $1.50 for 5 apples.

- This may be represented as the ratio 1.50 : 5.
- The number $\frac{1.50}{5} = 0.30$ is the value of the ratio.
- The unit price is then the value 0.30, to which we attach the units "dollars per apple." Thus the unit price can be written in any of the forms below.

$$0.30\ \frac{\text{dollars}}{\text{apple}} \qquad 0.30 \text{ dollars per apple} \qquad \$.30 \text{ per apple}$$

Ratios, Tables, and Graphs

A recipe calls for 2 parts lemon juice for every 3 parts water.

Lemon juice	2	4	6	8
Water	3	6	9	12

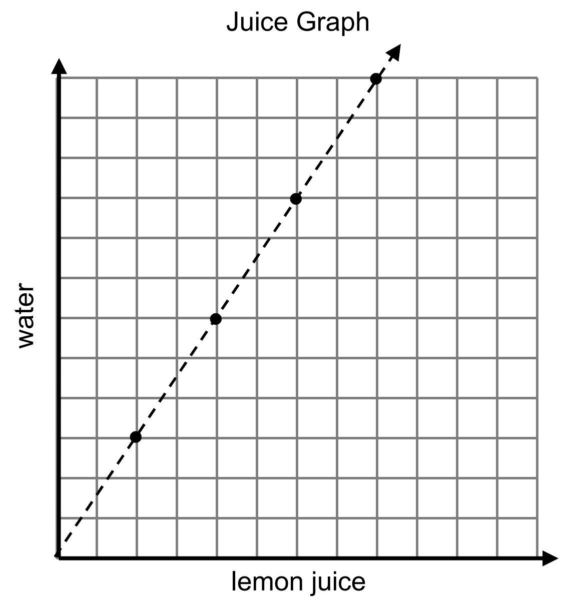
Data from tables can be graphed as ordered pairs in the coordinate plane.

We might choose the quantity listed first in the table (lemon juice) to be labeled on the horizontal axis, and the quantity listed second (water) to be labeled on the vertical axis.

Ratios of lemon juice to water are listed as ordered pairs below. Notice that these are equivalent ratios.

(2, 3) (4, 6)

(6, 9) (8, 12)



The graphed points lie on a straight line that passes through the origin. The part of this line in the first quadrant is a ray that emanates from the origin, (0, 0). That the graph passes through the origin is significant here, as this can be interpreted to mean that “0 units of lemon juice require 0 units of water.”

The ray contains the point (1, 1.5). This point is significant because it represents the rate of 1.5 parts water for each 1 part lemon, and 1.5 parts water per part lemon is the unit rate in this situation.

Representations of Proportional Relationships

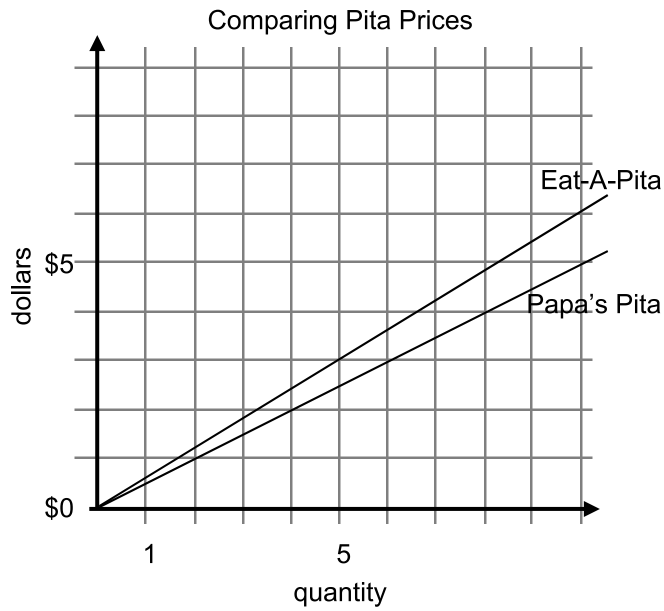
At Papa's Pitas, 2 pitas cost $1.00. At Eat-A-Pita, 5 pitas cost $3.00. Assuming a proportional relationship between the number of pitas and their cost, use multiple representations to explore which store offers the better buy for pitas.

Numbers (make a table)

Papa's Pitas	
# of pitas (x)	cost (y)
2	$1.00
4	$2.00
6	$3.00
8	$4.00
10	$5.00

Eat-a-Pita	
# of pitas (x)	cost (y)
5	$3.00
10	$6.00
15	$9.00
20	$12.00
25	$15.00

Pictures (make a graph)



Words (write sentences)

Based on the table, Papa's Pitas is the better buy.

At Papa's Pitas, you get 6 pitas for $3.00. This means the unit price (cost for one pita) is $0.50.

At Eat-A-Pita you only get 5 pitas for $3.00. This means the unit price (cost for one pita) is $0.60.

Symbols (write equations to relate the number of pitas to cost)

Papa's Pitas $y = 0.5x$

Eat-a-Pita $y = 0.6x$

Notice that $0.50 is the cost of one pita at Papa's Pita. This corresponds to the point (1, 0.5) on the graph.

Notice that $0.60 is the cost of one pita at Eat-A-Pita. This corresponds to the point (1, 0.6) on the graph.

The equations above are both in the form $y = mx$. This illustrates a proportional relationship because y is a multiple of x. Graphs of proportional relationships always lie on rays emanating from the origin.

MEASUREMENT

Metric Measurements	
Common metric units	**Examples (sizes approximate)**
Length	
1 millimeter (mm)	the thickness of a dime
1 centimeter (cm)	the width of a small finger
1 meter (m)	the length of a baseball bat
1 kilometer (km)	the length of 9 football fields
Capacity / Volume	
1 milliliter (mL)	an eyedropper
1 liter (L)	a juice carton
1 kiloliter (kL)	four filled bathtubs
Mass / Weight	
1 milligram (mg)	a grain of sand
1 gram (g)	a paperclip
1 kilogram (kg)	a textbook

U.S. Customary Measurements	
Common customary units	**Examples (sizes approximate)**
Length	
1 inch (in)	the length of a small paperclip
1 foot (ft)	the length of a sheet of paper
1 yard (yd)	the width of a door
1 mile (mi)	the length of 15 football fields
Capacity / Volume	
1 fluid ounce (fl oz)	a serving of honey
1 cup (c)	a small cup of coffee
1 pint (pt)	a bowl of soup
1 quart (qt.)	an engine oil container
1 gallon (gal)	a jug of milk
Mass / Weight	
1 ounce (oz)	a slice of bread
1 pound (lb)	a soccer ball
1 ton (T)	a walrus

Conversion Statements		
Length	**Capacity / Volume**	**Mass / Weight**
1 foot = 12 inches	1 cup = 8 fluid ounces	1 pound = 16 ounces
1 yard = 3 feet	1 pint = 2 cups	1 ton = 2,000 pounds
1 mile = 5,280 feet	1 quart = 4 cups	1 kilogram = 1,000 grams
1 kilometer = 1,000 meters	1 gallon = 4 quarts	
1 meter = 100 centimeter		1 kilogram $\approx$ 2.2 pounds
1 centimeter $\approx$ 0.4 inches	1 liter $\approx$ 1.06 quarts	
1 meter $\approx$ 39 inches		
1 kilometer $\approx$ 0.6 mile		
Area		
1 acre = 43,560 square feet		

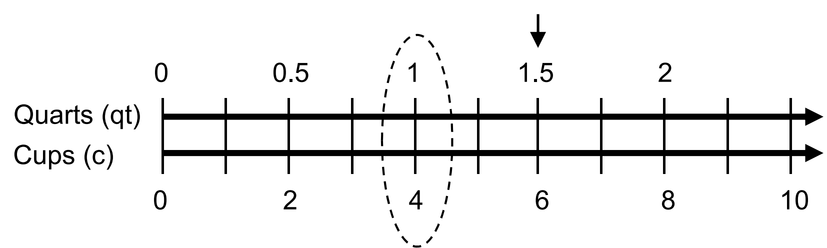
Conversions

Double number lines can be used to organize measurement conversion calculations.

Example: How many cups are in 1.5 quarts?

Create a double number line that shows 1 quart = 4 cups.

Then fill in other numbers on the line to answer the question.



There are 6 cups in 1.5 quarts.

Information from a double number line may also be organized into a table.

PERCENT

A <u>percent</u> is a number, expressed in terms of the unit 1% = $\frac{1}{100}$. The number n% is equal to $\frac{n}{100}$. It is the value of the ratio n : 100, and we can think of it as “n per 100.”

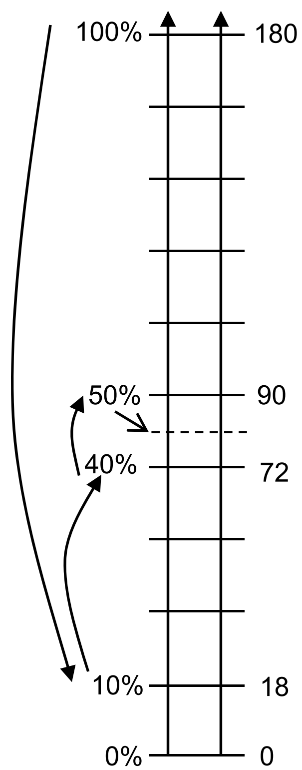
Some Fraction-Decimal-Percent Equivalents		
$\frac{1}{2}=\frac{50}{100}=0.5=50\%$ $\frac{1}{4}=\frac{25}{100}=0.25=25\%$ $\frac{3}{4}=\frac{75}{100}=0.75=75\%$ $\frac{5}{4}=\frac{125}{100}=1.25=125\%$ Conversion strategy: Think: $\frac{3}{4}\left(\frac{25}{25}\right)=\frac{75}{100}$	$\frac{1}{5}=\frac{20}{100}=0.2=20\%$ $\frac{2}{5}=\frac{40}{100}=0.4=40\%$ $\frac{3}{5}=\frac{60}{100}=0.6=60\%$ $\frac{4}{5}=\frac{80}{100}=0.8=80\%$ Conversion strategy: Think: $\frac{1}{5}=\frac{2}{10}=0.2$	$\frac{1}{8}=\frac{12.5}{100}=0.125=12.5\%$ $\frac{3}{8}=\frac{37.5}{100}=0.375=37.5\%$ $\frac{5}{8}=\frac{62.5}{100}=0.625=62.5\%$ $\frac{7}{8}=\frac{87.5}{100}=0.875=87.5\%$ Conversion strategy: Think: $\frac{1}{4}=\frac{25}{100}$ so half of $\frac{1}{4}=\frac{1}{8}=\frac{12.5}{100}$
$\frac{1}{10}=\frac{10}{100}=0.1=10\%$ $\frac{3}{10}=\frac{30}{100}=0.3=30\%$ $\frac{5}{10}=\frac{50}{100}=0.5=50\%$ Conversion strategy: Think: $\frac{3}{10}=\frac{30}{100}$ so 0.3 = 0.30	$\frac{3}{20}=\frac{15}{100}=0.15=15\%$ $\frac{13}{20}=\frac{65}{100}=0.65=65\%$ $\frac{19}{20}=\frac{95}{100}=0.95=95\%$ Conversion strategy: Think: 20 nickels in a dollar $\frac{1}{20}$ of a dollar is \$0.05	$\frac{1}{25}=\frac{4}{100}=0.4=4\%$ $\frac{16}{25}=\frac{64}{100}=0.64=64\%$ $\frac{9}{50}=\frac{18}{100}=0.18=18\%$ Conversion strategy: Think: 25(4) = 100, so $\frac{16}{25}\left(\frac{4}{4}\right)=\frac{64}{100}$

Using Multiplication, Division, and "Chunking" to Find Percents of Numbers	
Think	**Example**
Finding 100% of something is the same as finding all of it.	100% of \$80 = \$80 100%
Finding 50% of something is the same as finding one-half of it. This is the same as dividing by 2.	50% of \$80 = $\frac{1}{2}$(\$80) = \$40 \$80 ÷ 2 = \$40 50% | 50%
Finding 25% of something is the same as finding one-fourth of it. This is the same as dividing by 4.	25% of \$80 = $\frac{1}{4}$(\$80) = \$20 \$80 ÷ 4 = \$20 25% | 25% | 25% | 25%
Finding 10% of something is the same as finding one-tenth of it. This is the same as dividing by 10.	10% of \$80 = $\frac{1}{10}$(\$80) = \$8 \$80 ÷ 10 = \$8
Finding 1% of something is the same as finding one-hundredth of it. This is the same as dividing by 100.	1% of \$80 = $\frac{1}{100}$(\$80) = \$0.80 \$80 ÷ 100 = \$0.80
Finding 20% of something is the same as doubling 10% of it.	10% of \$80 = \$8 20% of \$80 = 2(\$8) = \$16
Finding 5% of something is the same halving 10% of it.	10% of \$80 = \$8 5% of \$80 = $\frac{1}{2}$(\$8) = \$4
Finding 15% of something is the same as adding 10% of it and 5% of it.	10% of \$80 = \$8, 5% of \$80 = \$4 15% of \$80 = \$8 + \$4 =\$12

Using Double Number Lines to Solve Percent Problems

Use double number lines to help you write equivalent fractions to represent each percent problem.

45% of 180 is what amount?

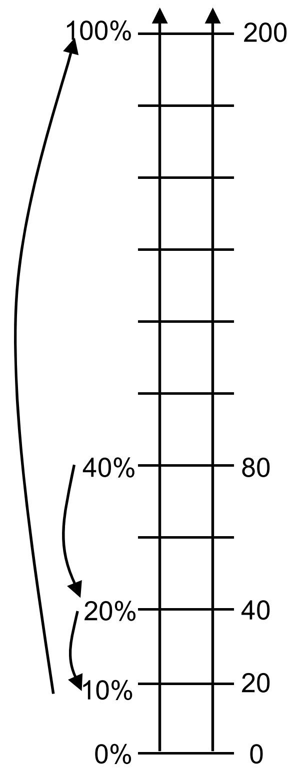


Think: If 100% → 180
then 10% → 18
and 40% → 72
and 50% → 90
so 45% → 81

$$\frac{45}{100} = \frac{81}{180}$$

45% of 180 is 81

40% of what amount is 80?

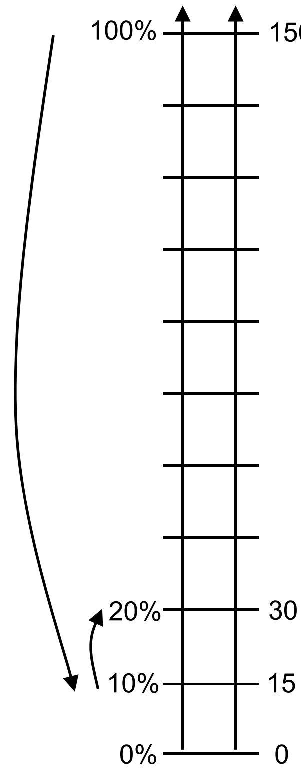


Think: If 40% → 80,
then 20% → 40
and 10% → 20
so 100% → 200

$$\frac{40}{100} = \frac{80}{200}$$

40% of 200 is 80

What percent of 150 is 30?



Think: If 100% → 150
then 10% → 15
so 20% → 30

$$\frac{20}{100} = \frac{30}{150}$$

20% of 150 is 30

Using Multiplication to Find Percents of Numbers

Some percents are challenging to find mentally. For example:

Finding 17% of something is the same as finding $\frac{17}{100}$, or 0.17 of it.

In this case, it may be easier to find the percent by using the definition of a percent of a number:

A <u>percent of a number</u> is the product of the percent and the number.

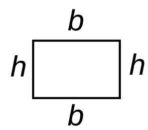
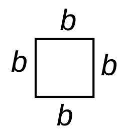
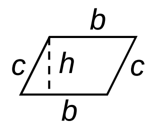
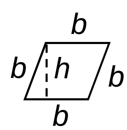
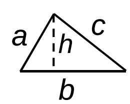
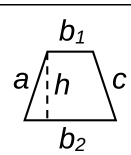
For example, to find 17% of $80:

<u>Use fractions</u> $\frac{17}{100} \bullet 80 = \frac{17 \bullet 80}{100} = \frac{1360}{100} = \frac{1360}{100} = 13\frac{3}{5}$

<u>Use decimals</u> $(0.17) \bullet (80) = 13.6$

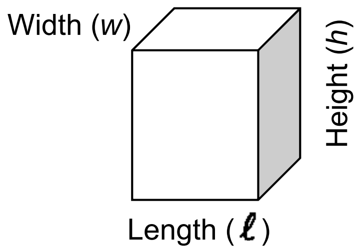
So 17% of $80 is $13.60

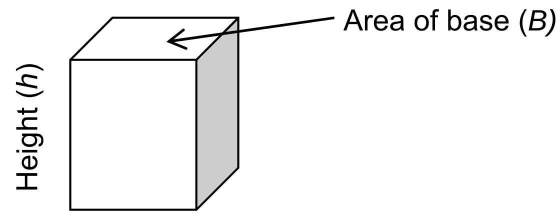
GEOMETRY

Summary of Perimeter and Area Formulas			
Shape/Definition	**Diagram**	**Perimeter**	**Area**
Rectangle a quadrilateral with 4 right angles		$P = 2(b + h)$ or $P = 2b + 2h$	$A = bh$
Square a rectangle with 4 equal sides		$P = 4b$	$A = b^2$
Parallelogram a quadrilateral with opposite sides parallel		$P = 2(b + c)$ or $P = 2b + 2c$	$A = bh$
Rhombus a quadrilateral with 4 equal sides		$P = 4b$	$A = bh$
Triangle a polygon with three sides		$P = a + b + c$	$A = \frac{1}{2}bh$
Trapezoid a quadrilateral with at least one pair of parallel sides		$P = a + b_1 + b_2 + c$	$A = \frac{1}{2}(b_1 + b_2)h$

The perimeter of a square is typically written as $P = 4s$, where s is the length of the side of the square. Similarly, perimeter of a rectangle is sometimes written $P = 2\ell + 2w$, where ℓ represents the length and w represents the width. For consistency, we use b in the formulas above when it refers to the length of a base. The consistent use of b makes the relationships among formulas more apparent.

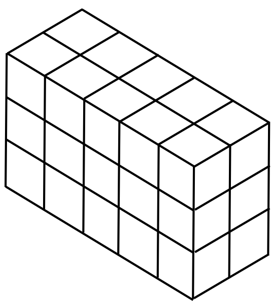
Right Rectangular Prisms

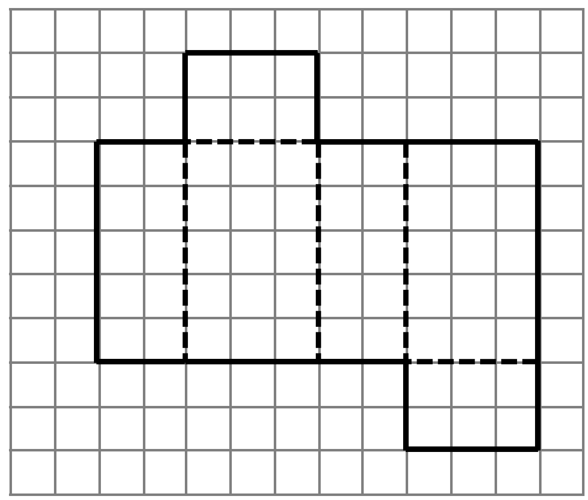




A right rectangular prism is identified by its length, width, and height.

The area of the base is the product of the length and width ($B = \ell w$).





The volume of a prism may be computed by counting layers of unit cubes. In the prism above, each layer has 10 cubes. There are 3 layers. The volume is 10 (3) = 30 cubic units.

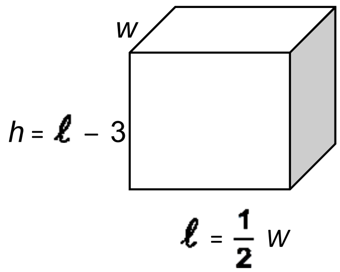
The surface area may be computed by creating a net that shows the areas of each face of the prism. In this prism there are two faces with dimensions 2×5, two faces with dimensions 3×2, and two faces with dimensions 3×5.

The surface area is $2(2 \times 5) + 2(3 \times 2) + 2(3 \times 5) = 20 + 12 + 30 = 62$ square units.

Volume	Surface Area
Multiply the area of the base (B) by the height. $V = \ell wh$ $V = Bh$	Find the area of each rectangular face. $SA = \ell w + \ell w + wh + wh + \ell h + \ell h$ $SA = 2\ell w + 2wh + 2\ell w$ $SA = 2(\ell w + wh + \ell h)$

Finding Surface Area and Volume of Right Rectangular Prisms

The width of a right rectangular prism is 20 centimeters. The length is half as long as the width. The height is 3 centimeters less than the length. Find its volume and surface area.



Define variables:

Width: $w = 20$ cm

Length: $\ell = \frac{1}{2}w = 10$ cm

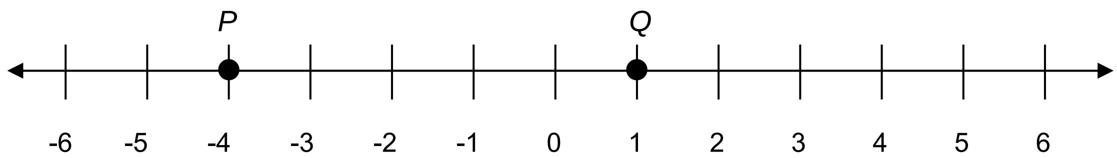
Height: $h = \ell - 3 = 10 - 3 = 7$ cm

Volume	Surface Area
Write a formula and substitute:	Write a formula and substitute:
$V = \ell wh$	$SA = 2(\ell w + wh + \ell h)$
$= 10 \bullet 20 \bullet 7$	$= 2(10 \bullet 20 + 20 \bullet 7 + 10 \bullet 7)$
$= 1{,}400$	$= 400 + 280 + 140$
	$= 820$
The volume is 1,400 cubic centimeters.	The surface area is 820 square centimeters.

THE NUMBER LINE

Integers on the Number Line

<u>Integers</u> (whole numbers and their opposites) may be represented on a horizontal or vertical number line.

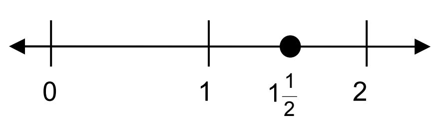
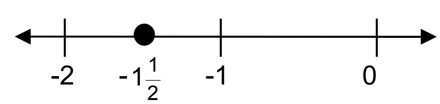


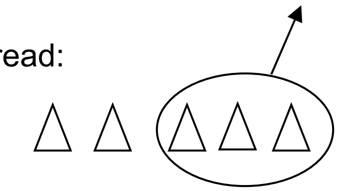
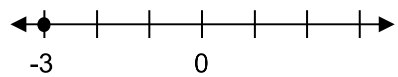
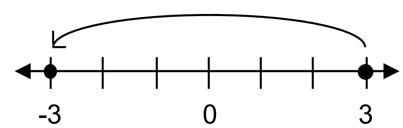
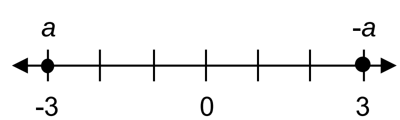
On the number line above, point *P* is located at -4, and point *Q* is located at 1.

Mixed Numbers and Their Opposites

<u>Rational numbers</u> are numbers that can be expressed as quotients of integers. These are numbers that can be expressed as $\frac{m}{n}$, where *m* and *n* are integers, and $n \neq 0$. Whereas fractions, decimals, and mixed numbers are greater than or equal to zero, the rational numbers include both positive and negative numbers.

Example: Both $\frac{3}{2}$ and $-\frac{3}{2}$ are rational numbers, but only $\frac{3}{2}$ is a fraction, while $-\frac{3}{2}$ is the negative (opposite) of a fraction.

	Mixed Number	**Negative (Opposite) of the Mixed Number**
	$1\frac{1}{2}$	$-1\frac{1}{2}$
Read as	"one and one-half"	"negative one and one-half" "the opposite of one and one-half"
The sum of	1 and $\frac{1}{2}$	-1 and $-\frac{1}{2}$
Number line representation		
As a fraction and its opposite	$\frac{3}{2}$	$-\frac{3}{2}$

Interpreting the Minus Sign	
Here are three ways to interpret the minus sign, along with some examples.	
Operation Interpretation When the minus sign is between two expressions, it means "subtract the second expression from the first."	Example: 5 – 3 The phrase "5 minus 3" can be read:  • 5 take away 3 • The difference between 5 and 3 • Subtract 3 from 5
Geometric Interpretation In front of a number or variable, a minus sign means "opposite." Geometrically, minus can be thought of as a reflection or mirror image. In this case, we are reflecting the number line through zero.	Example: -3 The phrase "minus 3" can be read: • Negative 3  Pictorially, this is a location on the number line that is 3 units left of zero. • Opposite of 3  This is the value you get by first locating 3 on the number line, and then locating that same distance on the opposite side of zero. Geometrically, minus can be thought of as a reflection or mirror Image. In this case, the reflection of 3 through zero is -3.
Algebraic Interpretation The minus sign is used to show additive inverses. The identity $a + (-a) = 0$ means that $-a$ is the additive inverse of a. It is what we add to a to get 0.	Example: If $a = -3$, then $-a = 3$ The statement, "If a is equal to minus 3, then minus a is equal to 3" can be read: • If a is equal to the opposite of 3, then the opposite of a is equal to 3.  Be careful not to read the variable $-a$ as "negative a" because the variable may not represent a number less than zero. However, reading $-a$ as "the opposite of a" is appropriate. For example, $-(-3) = 3$ may be read "the opposite of the opposite of 3 is 3.

Distance and Absolute Value

The <u>absolute value</u> of a number is its distance from zero on the number line.

A distance 25 units in the positive direction from zero is written $|+25| = 25$.

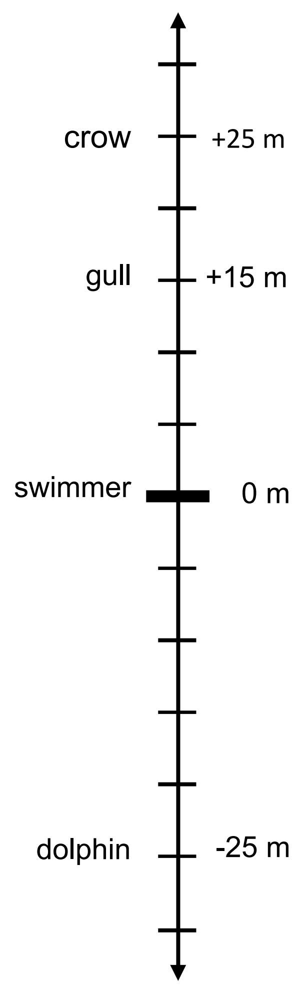
A distance 25 units in the negative direction from zero is written $|-25| = 25$.

Note that if $x \geq 0$, then $|x| = x$, while if $x < 0$, $|x| = -x$. In other words, the absolute value of a positive number is equal to the number itself, while the absolute value of a negative number is the opposite of the number. The absolute value of zero is simply zero.

<u>Distance</u> is always greater than or equal to zero.

<u>Elevation</u> relative to sea level is measured vertically from sea level. Elevation may be positive, negative, or zero.

The vertical number line to the right represents elevations from 25 meters below sea level (-25 m) to 25 meters above sea level (+25 m).



What	Elevation	Distance from zero (sea level)	Absolute value equation for distance from sea level
crow	+25 m	25 m	$|25| = 25$
gull	+15 m	15 m	$|15| = 15$
swimmer	0 m	0 m	$|0| = 0$
dolphin	-25 m	25 m	$|-25| = 25$

Here are some true statements about elevation:

- $(15 > -25)$ The gull is at a higher elevation than the dolphin.
- $(0 < 25)$ The swimmer is at a lower elevation than the crow.

Here are some true statements about absolute value:

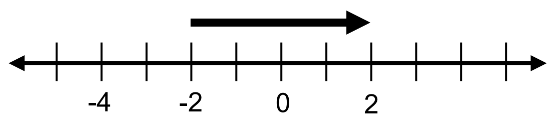
- $(|-25| = |25|)$ The dolphin and the crow are the same distance from 0.
- $(|-25| = |25|)$ The dolphin and the crow are both 25 meters from sea level.
- $(|-25| > |15|)$ The dolphin is farther from sea level than the gull.

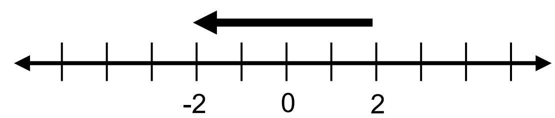
A Vector Model

Numbers can be represented by arrows on a number line. Arrows represent <u>distance</u> (or length) and <u>direction</u>. On a number line, the sign of a number is represented by its direction. The absolute value of a number is represented by the length of the arrow from head to tail.

The first arrow represents 4. It starts at -2 and ends at 2. Its length is 4. If we slide the arrow along the number line so that its tail is at 0, the point of the arrow will be at 4.

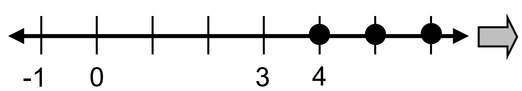
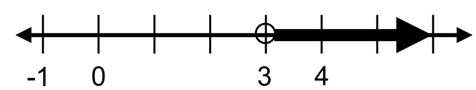
The second arrow represents -4. It starts at 2 and ends at -2. Its length is 4. If we slide the arrow along the number line so that its tail is at 0, the point of the arrow will be at -4.





Graphing Inequalities on the Number line

Here are some ways to represent solutions to inequalities on a number line.

Integer values on the number line that satisfy the inequality $n > 3$.	**All numbers** on the number line that satisfy the inequality $x > 3$.
	
The dots represent integers that are greater than 3. These are <u>integer solutions</u> to the inequality. The arrow indicates that all the integers to the right on the number line are solutions.	The "open dot" indicates that $x = 3$ is not included in the solution set. That is, it is not a <u>solution</u> to the inequality $x > 3$. The arrow indicates that all numbers to the right on the number line are solutions.

It is common in mathematics to use the variable n to represent an integer and x to represent a real number (i.e. any type of number, not just an integer). Sometimes the variable used can be an indication of which type of diagram should represent the solution to the inequality.

THE COORDINATE PLANE

The Coordinate Plane

A (Cartesian) coordinate plane is determined by a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin.

Points are located using ordered pairs (x, y).

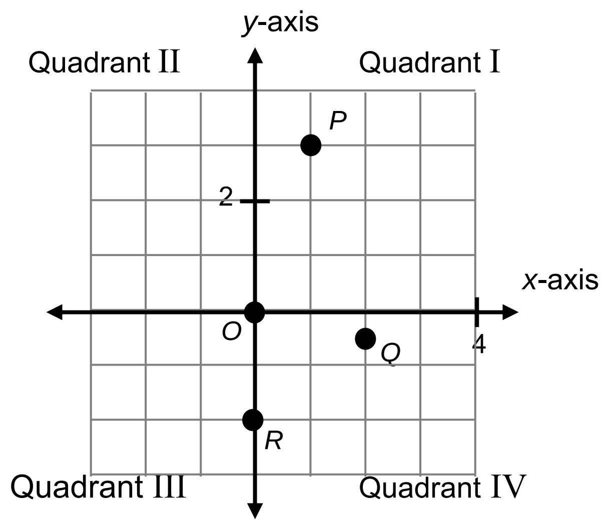
- The first number (*x*-coordinate) is its location on the horizontal number line through (x, y). The point is to the right of the *y*-axis if $x > 0$, to the left of the *y*-axis if $x < 0$, and it is on the *y*-axis if $x = 0$. The absolute value of the *x*-coordinate is the distance of the point to the *y*-axis.

- The second number (*y*-coordinate) is its location on the vertical number line through (x, y). The point is above the *x*-axis if $y > 0$, below the *x*-axis if $y < 0$, and on the *x*-axis if $y = 0$. The absolute value of the *y*-coordinate is the distance of the point to the *x*-axis.

- The point of intersection of the two axes is called the origin, and we identify it with the ordered pair (0, 0).

The axes (plural of axis) divide the plane into four regions, called quadrants. By convention, we number the quadrants using Roman numerals I-IV, starting with the upper right quadrant (first quadrant) and moving counterclockwise to the lower right quadrant (fourth quadrant). The axes may be considered as boundary lines and are not part of any quadrant.

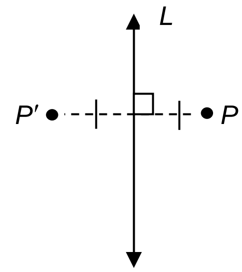
Point	Coordinates	Location
O	(0, 0)	origin
P	(1, 3)	Quadrant I
Q	$(2, -\frac{1}{2})$	Quadrant IV
R	(0, -2)	*y*-axis



Reflections

A reflection of the plane through a line L is a rule that sends each point P to its “mirror image” relative to L.

The line L is referred to as the line of symmetry of the reflection.

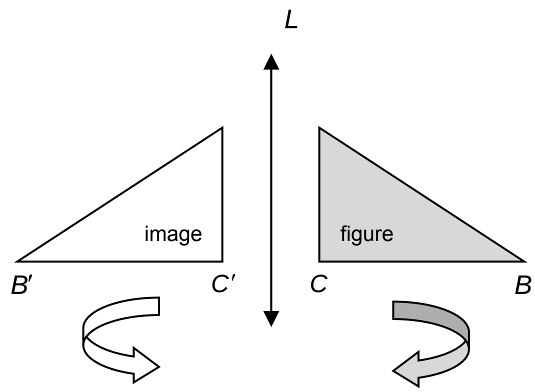


One way to see if two points are reflections of one another is by folding a sheet of paper. When the paper is folded along the line of symmetry, a point will land on its reflection.

Paper folding shows that if P' (in the diagram to the right) is the reflection of P through L, then P is the reflection of P' through L. In other words, if we reflect twice through L, the point P will end up at P, right where we started.

Here are two important properties of reflections:

Reflections preserve distance. Consider a figure (such as triangle ABC to the right), and the image of the figure (such as triangle $A'B'C'$ to the right) reflected through L. The distance between any two points in the original figure (e.g. the distance from A to B) is the same as the distance between the corresponding image points (e.g. the distance from A' to B').



Reflections reverse orientation. If we track around the triangle in a clockwise direction (e.g. A to B to C), the corresponding reflected points track around the image triangle in a counterclockwise direction (e.g., A' to B' to C').

9 7 8 1 6 1 4 4 5 0 2 3 8